



Effect of equation of state and composition on relativistic flows

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Abstract. The thermal state of the fluid is governed by the ratio of the thermal and the rest energy. This brings the composition of the fluid into the picture. The fluid composed of lighter particles (e.g: electron-positron pair plasma) at same temperature, is more relativistic compared to fluids with finite baryon loading, but this is not necessarily true when baryon poor transonic fluid are compared with each other. It could be shown that the transonic pair-fluid is the least relativistic. This result has far reaching consequences on accreting flows around compact objects and are expected to have similar effect on relativistic outflows and explosive events as well.

Keywords : accretion, accretion discs — Jets, outflows and bipolar flows — black hole physics — hydrodynamics — Transonic fluids — shock waves — relativity

1. Introduction

Relativistic fluids are expected in accretion discs around compact objects, relativistic jets around AGNs, micro-quasars and in Gamma Ray Bursts (GRBs). A fluid is coined relativistic if its bulk velocity is comparable to the speed of light (c). Terminal speeds of jets around micro-quasars and AGNs are observed to be close to c (Mirabel & Rodriguez 1994; Zensus et al. 1995). In case of GRBs, ultra-relativistic jets with Lorentz factors $\gamma > \text{few} \times 100$ are invoked to explain observations (Meszaros 2002).

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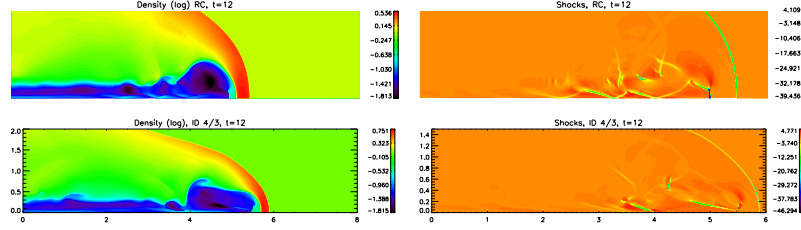


Figure 1. 2D Slab jet simulation in Cartesian space-coordinates, resolution 1024×256 , $r_b = 12$ cells, $v_b = 0.95$, $\rho_b/\rho_a = 0.1$, and $p = 0.01$. Density contours for RC (Upper left) and for ID EoS (Lower left) in log scale. The suffix ‘a’ and ‘b’ stands for ambient medium quantities and jet beam quantities, respectively. Projection of shock surfaces for RC EoS (upper right) and for ID EoS (Lower right). All the figures are for the same snap shot $t=12$.

General relativity requires that matter plunges onto the black hole with the speed of light. Moreover, analysis of high energy photons received from micro-quasars and AGNs, gives evidence of very high proton temperatures $T_p \gtrsim 10^{12} K$ and electron temperatures $T_e \gtrsim 10^9 K$. If the thermal energy of the fluid is comparable to its rest energy ($kT/mc^2 \gtrsim 1$) then the fluid is considered to be *thermally relativistic* and the thermal state is represented by the adiabatic index $\Gamma \sim 4/3$. Conversely, for $kT/mc^2 \ll 1$, the fluid is called thermally non-relativistic ($\Gamma = 5/3$). Here k and m are the Boltzman constant and the mass of the fluid particles, respectively. It is clear that the adiabatic index of the fluid depends on the thermal energy and is not a constant. Constancy of Γ is due to the non-relativistic treatment of the motions of the gas particles. A proper relativistic treatment results in a relativistic equation of state (EoS) of the fluid, and a temperature dependent Γ with correct asymptotic limits (Chandrasekhar 1938; Synge 1957). The relativistic EoS were applied to relativistic flows (Blumenthal & Mathews 1976; Fukue 1987; Meliani et al. 2004; Mignone et al. 2005; Ryu et al. 2006; Chattopadhyay & Ryu 2009; Chattopadhyay & Chakrabarti 2011), although the general trend had been to use the fixed Γ EoS (hereafter abbreviated as ID for ideal EoS).

In this paper, we present the applications of relativistic thermodynamics onto astrophysical fluids, and show that consideration of relativistic EoS produces significantly different result from that of the ID EoS. Moreover, we will further show that consideration of the composition of the fluid influences the flow solutions both qualitatively and quantitatively.

2. Numerical simulation with single species fluid

Assuming single species fluid and relativistic kinetic theory of gas, the EoS is computed by Chandrasekhar (1938); Synge (1957), which is dependent on the ratios of modified Bessel functions of second and third kind. A very good algebraic approxi-

mation (abbreviated as RC) was proposed by Ryu et al. (2006), and is given by

$$e = \rho h - p = \rho + p \left(\frac{9p + 3\rho}{3p + 2\rho} \right). \quad (1)$$

Using RC EoS, a numerical simulation code was developed by Ryu et al. (2006) following the TVD formalism (Harten 1983). The relativistic equations of motion are

$$T^{\mu\nu}_{;\nu} = 0 \quad \text{and} \quad (nu^\nu)_{;\nu} = 0, \quad (2)$$

The complete eigenstructure for Eqs 1,2 were presented in Ryu et al. (2006). The first simulation is for 2D slab jet with RC and ID EoS (see, caption of Fig. 1 for details). Although the fluid with different EoS are launched with the same initial conditions, the evolution of the jets are different, and more importantly, the shock structures generated by the two jets due to the interaction with the ambient medium, are completely different. Such remarkable difference in shock structures and overall flow variables, will definitely generate different spectra.

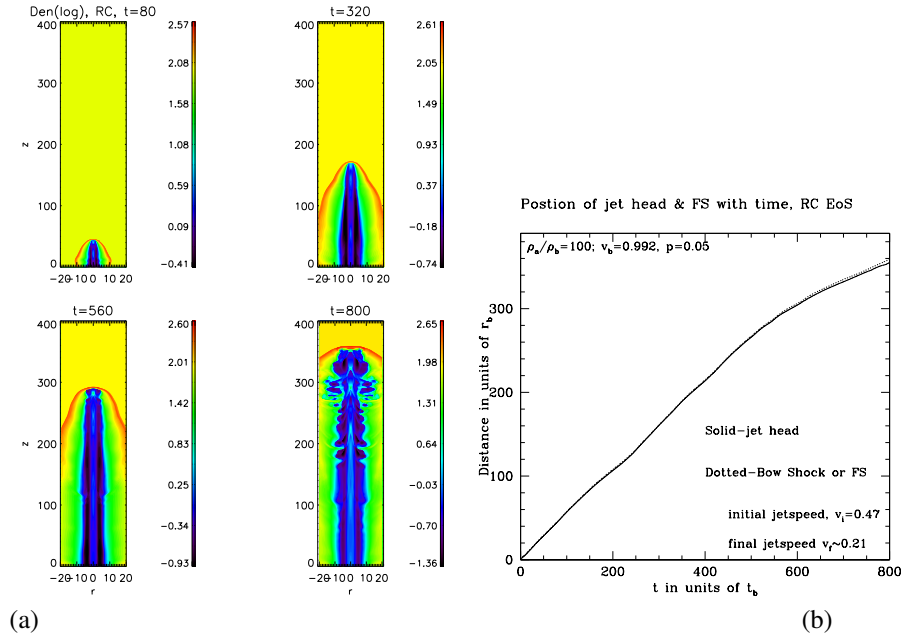


Figure 2. 2D cylindrical axisymmetric jet simulation, resolution 240×4800 , $r_b = 24$ cells, $v_b = 0.992$, $\rho_b/\rho_a = 10^{-2}$, and $p = 0.05$. (a) Density contours (log scale) in the $r - z$ plane for RC EoS, at four time snaps. The unit of length is initial jet beam width r_b and that of time is $t_b = r_b/c$ or the light crossing time of r_b . (b) The time evolution of the forward shock (FS) and the jet head for the above case.

In Fig. 2a, four snap shots of the evolution of density contours for 2D cylindrical axisymmetric jet simulation with RC EoS are plotted. In Fig. 2b, the forward shock and

jet head is plotted with time. As long as the jet accelerates, very little flow structure is formed ($t \lesssim 500 t_b$), but as the jet decelerates, *i.e.*, as the beam loses its kinetic energy, the back flowing material interacts with the beam itself and many structures are seen to be formed.

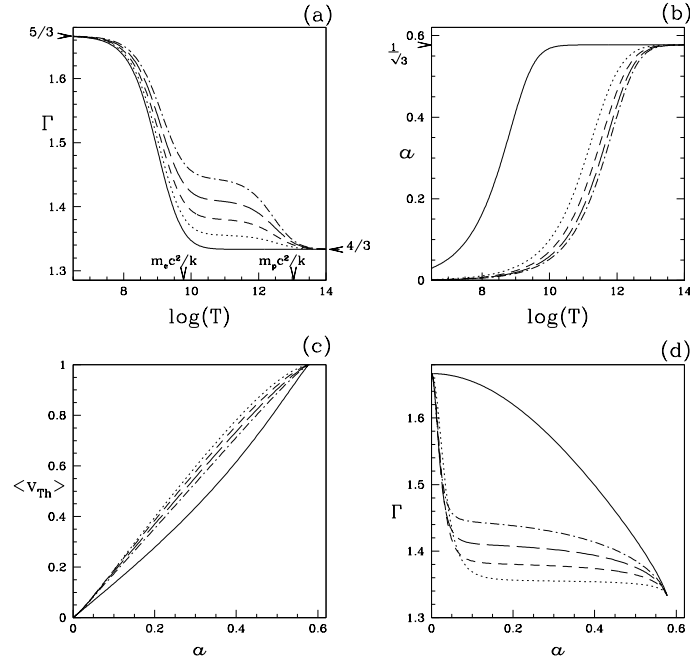


Figure 3. (a) Γ , and (b) sound speed a are plotted with $\log(T)$, (c) thermal velocity $\langle v_{Th} \rangle$ and (d) Γ with a , for $\xi = 0$ *i.e.*, $e^- - e^+$ (solid), $\xi = 0.25$ (dotted), 0.5 (dashed), 0.75 (long dashed), and 1.0 *i.e.*, $e^- - p^+$ (dashed-dotted).

3. Multi-species fluid

As has been noted in the previous section that a fluid is considered thermally relativistic if $kT/mc^2 \gtrsim 1$, therefore, it is not only the T alone but the ratio T/m , which determines the thermal state of a fluid. Fluids composed of similar particles (independent of its mass) are thermodynamically similar and hence, their time evolution will be similar under same physical conditions. However, a fluid composed of a mixture of dissimilar particles will be different. Assuming a fluid composed of electron (e^-), positron (e^+) and protons (p^+), where each species of the fluid obey RC EoS, then the proper energy density is given by Chattopadhyay & Ryu (2009)

$$e = n_e m_e f, \quad (3)$$

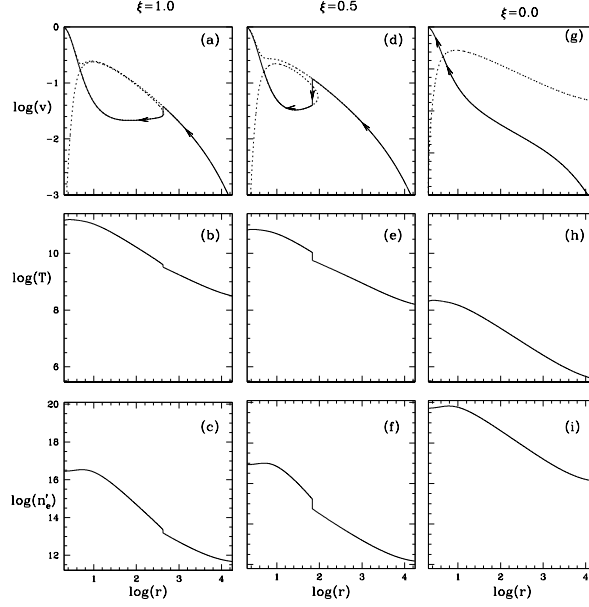


Figure 4. Constants of motion $\{\mathcal{E}, \lambda\} = \{1.00009, 3.4\}$. The first column is for $\xi = 1.0$ (Figs: a, b, c); the second for $\xi = 0.5$ (Figs: d, e, f) and the third for $\xi = 0.0$ (Figs: g, h, i). Each row represents flow variables $\log(v)$ (Figs: a, d, g); $\log(T)$ (Figs: b, e, h); and $\log(n'_e)$ (Figs: c, f, i); where $n'_e = n_{e^+} + n_{e^-}$. Vertical jumps represent standing shocks. For $\xi = 1.0$ $x_s = 423.46$, and $\xi = 0.5$ $x_s = 68.78$. Solid curve with arrow, the top row show solutions chosen by the shocked accreting matter from all the associated solutions (dotted).

where,

$$f = (2 - \xi) \left[1 + \Theta \left(\frac{9\Theta + 3}{3\Theta + 2} \right) \right] + \xi \left[\frac{1}{\eta} + \Theta \left(\frac{9\Theta + 3/\eta}{3\Theta + 2/\eta} \right) \right]. \quad (4)$$

In Eqn. 4, $\Theta = kT/m_e c^2$ where, k , and m_e are the Boltzman's constant and electron rest mass, respectively. Here, ξ is the ratio of the number densities of protons and electrons, so $\xi = 0$ is electron-positron plasma or $e^- - e^+$, and $\xi = 1$ is electron-proton plasma or $e^- - p^+$. Moreover, η is the ratio between an electron and a proton mass. The definition of the adiabatic index $\Gamma = 1 + 1/N$, where $2N = df/d\Theta$. In Fig. 3a, Γ is plotted as a function of T . The thermal state of the fluid distinctly depends on composition parameter ξ and T . At a given T , the fluid composed of lightest particles ($e^- - e^+$) is more relativistic *i.e.*, Γ is the lowest, but Γ increases with ξ . For non-relativistic temperatures $T \lesssim 10^7 \text{K}$, and ultra relativistic temperatures $T \gtrsim 10^{13} \text{K}$ the dependence of the fluid thermodynamics on ξ is nominal.

In Fig. 3b, the local sound speed $a = \Gamma p/(e + p)$ is plotted with $\log(T)$. At same T , the fluid composed of lighter particles have higher sound speed. At the same thermal energy the fluid composed of lightest particles will be the most relativistic. The thermal velocity $\langle v_{Th} \rangle$ is plotted with a in Fig. 3c. The thermal velocity

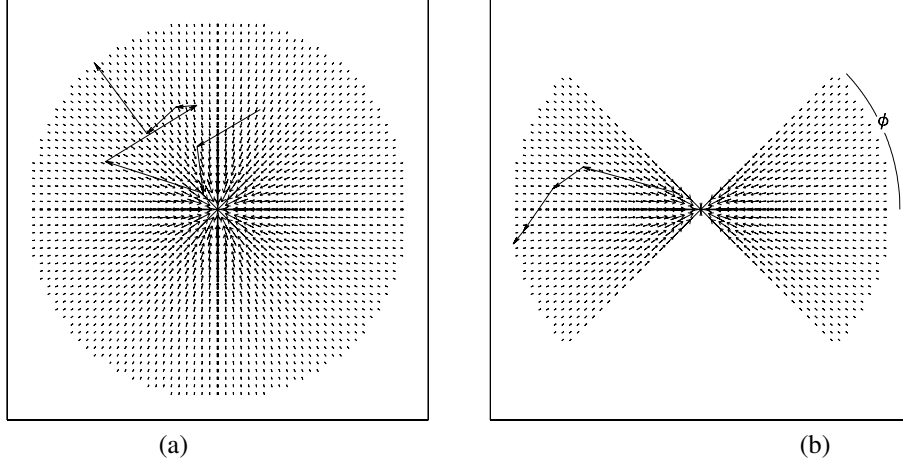


Figure 5. 2D projection of Monte-Carlo Simulation: (a) Velocity vector and path of a single photon for spherical accretion. (b) Same for a conical flow. The composition is $\xi = 1$. The simulation box is $500r_g$.

$\langle v_{Th} \rangle$ can be easily expressed in terms of Θ and ξ from the following relationship,

$$e = \rho c^2 \langle \gamma_{Th} \rangle = \sqrt{\langle q_{Th} \rangle^2 c^2 + \langle m \rangle^2 c^4} = n_e m_e c^2 f, \quad (5)$$

where, $\langle \gamma_{Th} \rangle$, $\langle q_{Th} \rangle$ and $\langle m \rangle$ are the average particle Lorentz factor, particle momentum and particle rest mass in the fluid rest frame, respectively. Clearly, v_{Th} is minimum for the $e^- - e^+$ fluid (solid), but is not maximum for $e^- - p^+$ (dashed-dotted). For a given value of a , v_{Th} is maximum for a fluid of $\xi \sim 0.24$. The relation between Γ and a is plotted in Fig. 3d. At a given a , Γ is the least relativistic for $e^- - e^+$, and most relativistic for a fluid of $\xi \sim 0.24$. Figure 3c shows that at a given a , the momentum transferred by the lighter particles in the fluid frame is far less than the heavier particles. As a result, T of $e^- - e^+$ fluid is much less compared to that of the fluid with $0 < \xi \leq 1$. However, with the increase of ξ , the rest energy per oppositely charged particles of the fluid increases too. Hence, a fluid becomes most relativistic at the optimal value of $\xi \sim 0.24$.

3.1 Single temperature, adiabatic flow onto black holes

Equation 2, when applied for steady, radial flow onto Schwarzschild black holes, admits critical point relations (Chakrabarti 1990; Chattopadhyay & Ryu 2009), which are also the sonic points, and are given by,

$$v_c^2 = a_c^2; \quad a_c^2 = \frac{1}{(2r_c - 3)}, \quad (6)$$

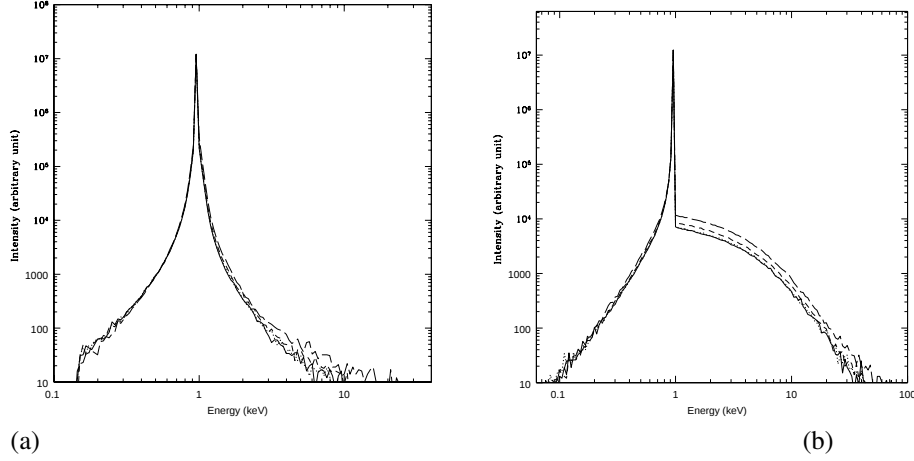


Figure 6. Monte-Carlo Simulation Spectrum for $e^- - e^+$ (a) and $e^- - p^+$ (b). Curves are for spherical flow or $\phi = 90^\circ$ (solid), $\phi = 60^\circ$ (dotted), $\phi = 30^\circ$ (dashed), $\phi = 10^\circ$ (long dashed). ϕ is the angle shown in Fig. 5b.

where, the subscript ‘c’ denotes the quantities evaluated at the critical point $r = r_c$, and $v^2 = -u_r u^r / u_t u^t$ is the radial three velocity. A black hole accretion is definitely transonic, so formation of a sonic point is a necessary feature of black hole accretion. Two transonic solutions can be compared at the same sonic point, or having the same specific energy at infinity, *i.e.*, same $\mathcal{E}|_\infty$. Flows with same $\mathcal{E}|_\infty$ implies comparison of flows with same a_∞ . Moreover, flows with same r_c , implies flows with same a_c . Hence according to Figs 3b-3d, a transonic $e^- - e^+$ fluid is expected to be the least relativistic, and of the least temperature of all. Chattopadhyay & Ryu (2009) showed it is indeed so. Since hot, rotating relativistic flows around black holes are supposed to form multiple critical points and shock, it should be interesting to see how composition affects the solutions of such flows. In Figs 4a-i, various flow variables are plotted, which show that for the same flow parameters $\{\mathcal{E}, \lambda\} = \{1.00009, 3.4\}$, shock and multiple sonic points may form for flow with some baryon content, while for purely leptonic flow neither multiple sonic point form, nor shocks form (Chattopadhyay & Chakrabarti 2011). Clearly, if the flow starting with same outer boundary condition can have such a widely different solutions, then the radiative properties will be distinctly different.

3.2 Monte Carlo simulation and spectrum

As a representative case, let us compare the radiative properties of fluids of two different compositions *i.e.*, $e^- - e^+$ and $e^- - p^+$, but starting with same energy at infinity ($\mathcal{E}_\infty = 1.001$). The seed photons are mono-energetic photons of 1keV, and are randomly injected through out the flow. Various flow geometries from purely spherical

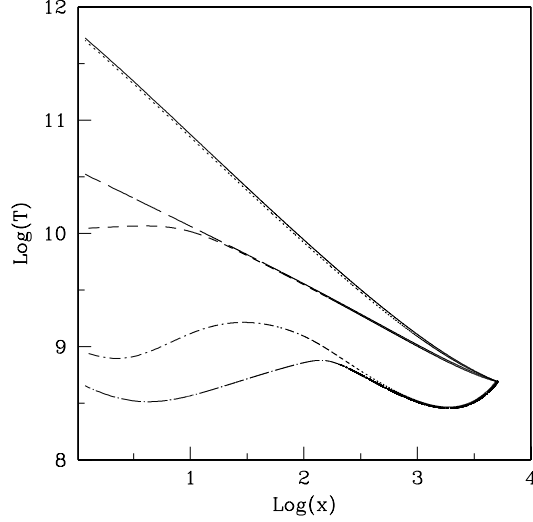


Figure 7. Two temperature solution for $e^- - p^+$ fluid. T_p (solid) and T_e (long dashed) for adiabatic flow, T_p (dotted) and T_e (short dashed) when $\dot{M} = 0.1\dot{M}_{\text{Edd}}$, and T_p (dot-short dashed) and T_e (dot-long dashed) when $\dot{M} = 2.5\dot{M}_{\text{Edd}}$. The cooling process is Bremsstrahlung and Comptonization (inverse).

(Fig. 5a) to conical shaped (Fig. 5b) accretion has been considered. The spectrum for $\xi = 0$ fluid is presented in Fig. 6a, while that of $\xi = 1$ in Fig. 6b. The spectra from all the models (see Figs 5a-b) of accretion for $e^- - e^+$ fluid is soft, and almost indistinguishable. However, the spectra of the $e^- - p^+$ fluid is hard and depends on the opening angle of the flow. This is expected since $e^- - p^+$ fluid is hotter than $e^- - e^+$ fluid for the same outer boundary condition and hence electrons in $e^- - p^+$ fluid more efficiently transfer energy to photon due to inverse Comptonization. In presence of realistic cooling processes, the flow may become two temperature. We have also solved the two temperature solution for the $e^- - p^+$ fluid in presence of bremsstrahlung and its Comptonization. The dependence of the flow solution on the accretion rate is quite distinct. In Fig. 7, the proton (T_p) and electron (T_e) temperatures are plotted for adiabatic flow, and flows with cooling process for accretion rates *i.e.*, $\dot{M} = 0.1$ & $2.5 \dot{M}_{\text{Edd}}$. The flows have the same outer boundary condition *i.e.*, $\mathcal{E} = 1.000136$ at $r = 10^4 r_g$.

4. Conclusion

It has been shown in the paper that relativistic equation of state is very important to study relativistic astrophysics of jets or accretion disc. It has also been shown that the composition of the fluid is extremely important too. Purely leptonic flow onto black holes do not show high energy phenomena like shocks. Since transonic pair plasma fluid is not very relativistic, therefore spectra from such flow are softer. If one

considers fluid which are a mixture of leptons and hadrons, then single temperature solution in absence of ohmic dissipation, may be an over simplified assumption.

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